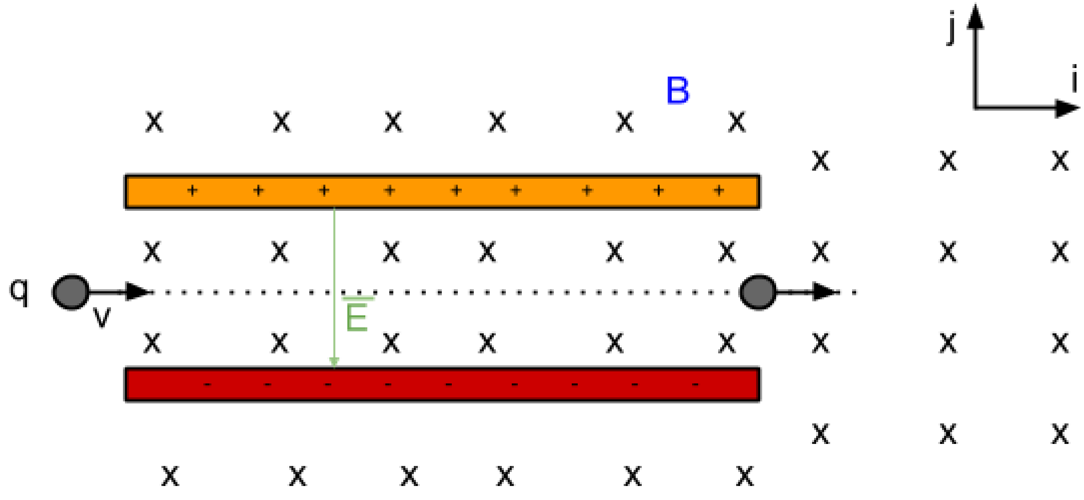
Electromagnetic Force Example (force on a charge due to both electric and magnetic fields)

A positive charge (q) initially traveling at a constant velocity v enters the region between a parallel plate capacitor which has an external, homogeneous magnetic field flowing into the page.



(1) What should be the strength B of the magnetic field to ensure the particle continues moving along a completely horizontal path, without colliding with the capacitor's plates?

(2) Describe the path of the particle as it exits the region of the parallel plate capacitor.

<u>Solution</u>

Step 1: Compute the magnitudes of the forces acting on the charged particle as it enters the region between the plates.

Two kinds of forces will be acting on the particle: (i) an electric force due to the electric field between the positive and negative plates, and (ii) a magnetic force due to the magnetic field interacting with the moving charge. The equation for each is provided below:

(i) $\overrightarrow{F_{electric}} = q\vec{E}$

(ii) $\overrightarrow{F_{magnetic}} = q\vec{v} \times \vec{B}$

Step 2: Determine the directions of the electric and magnetic forces on the charge.

Since this is a positive charge, it will be attracted towards the bottom, negative plate. As such, $\overrightarrow{F_{electric}}$ is pointing downwards.

Using the right-hand rule for the cross product between $\vec{v}$ and $\vec{B}$, we see that the magnetic force on the particle is pointing upwards.

Therefore the net force on the particle is: $\overrightarrow{F_{net}} = \overrightarrow{F_{electric}} + \overrightarrow{F_{magnetic}} = -qE\, j \; + \; qvB\, j$

Step 3: Determine the magnitude of the magnetic field.

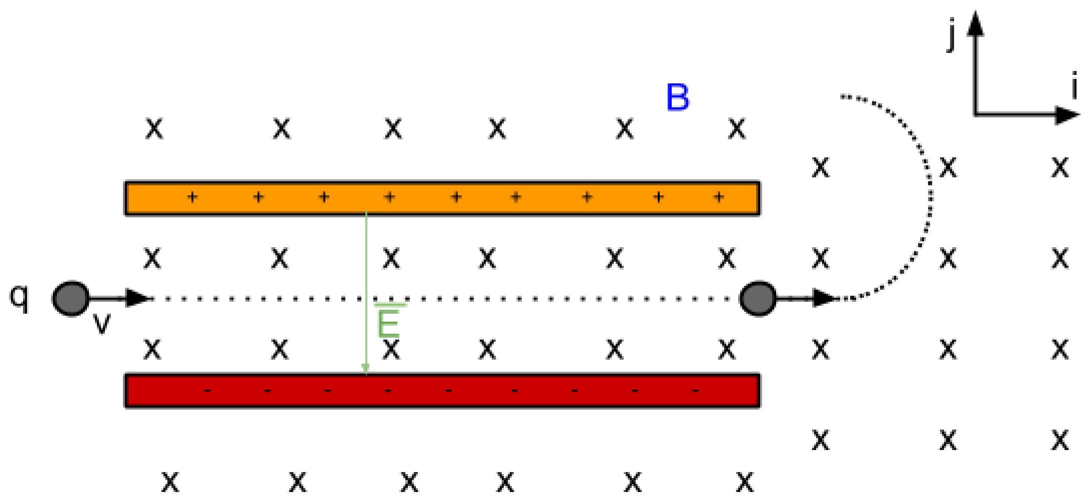
For the charge to not change its path, its net force in a direction perpendicular to the direction of motion should be zero. i.e. $\overrightarrow{F_{net}} = (-qE + qvB)j = 0$

Isolating B from this equation, we obtain the **answer to part (1):** $\boldsymbol{B} = \frac{\boldsymbol{E}}{\boldsymbol{v}}$

Step 4: Determine the net force on the particle just as it exits the region between the plates.

Since no electric field exists outside the plate region, the particle no longer experiences the downward $\overrightarrow{F_{electric}}$. As a result, it has an upward net force: $\overrightarrow{F_{net}} = \overrightarrow{F_{magnetic}} = qvB\ j$

Now, the particle has a constant velocity in the i-direction, as well as a net force in the perpendicular, upward direction. These result in an upward circular motion, as illustrated below.



Answer to part (2): the particle will move upward and in a circular motion.