Capacitance Example (for a parallel-plate capacitor)

A parallel plate capacitor has a plate separation of 0.8 cm and a plate area of 20 cm^2. The capacitor is connected to a battery that provides 5 V. The battery is then disconnected, and a piece of glass ($\kappa = 5.3$) filling the region between the parallel plates is inserted. Calculate the capacitance, the charge on the plates, and the potential difference between the plates after the dielectric is inserted.
Given: $\varepsilon_o = 8.854 * 10^{-12}$ C^2/Nm^2.

<u>Solution</u>

Step 1: Calculate the capacitance of the capacitor in absence of any dielectric (assuming air, or vacuum).

Using the equation: $C_0 = \frac{\epsilon_0 A}{d} \Rightarrow \quad C_0 = \frac{8.854 \cdot 10^{-12}\ C^2/Nm^2 \cdot 20 \cdot 10^{-4}\ m^2}{0.8 \cdot 10^{-2}\ m} = 2.214 \cdot 10^{-12}\ F$

Step 2: Calculate the charge stored on this capacitor while it is connected to the battery.

$$Q_0 = C_0 V_{battery} \quad \Rightarrow \quad Q_0 = 2.214 \cdot 10^{-12} \cdot 5 = 11.07 \cdot 10^{-12}\ C$$

Once the battery is disconnected, the charge on the plates of the capacitor does not change, even when the dielectric between the plates is changed. This is because the power source that pushes charge onto the plates is no longer connected, and the open circuit does not allow the charge on the plates to escape. Hence, $Q = Q_0$.

Step 3: Calculate the new capacitance, once the dielectric is inserted between the plates.

$$C = \frac{\kappa \epsilon_0 A}{d} \Rightarrow \quad C = \kappa C_0 = 5.3 \cdot (2.214 \cdot 10^{-12}) = 11.73 \cdot 10^{-12} F$$

Step 4: Calculate the new voltage difference across the capacitor.

$$Q = CV \quad \Rightarrow V = \frac{Q_0}{C} = \frac{11.07 \cdot 10^{-12}}{11.73 \cdot 10^{-12}} = 0.94\ V$$

Answer: $Q = Q_0 = 11.07 \cdot 10^{-12}\ C,\ \ C = 11.73 \cdot 10^{-12}\ F\ ,\ V = 0.94\ V$