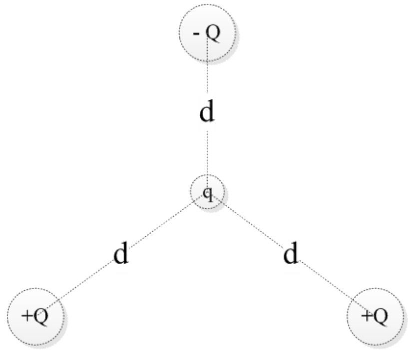
Force Hard Example (determine the force due to three charges using Coulomb's law):

Calculate the net force experienced by the charge q due to the charges in the following diagram, where q = -1 $* 10^{-6}$ C, Q = 2 $* 10^{-6}$ C and d = 10 cm.

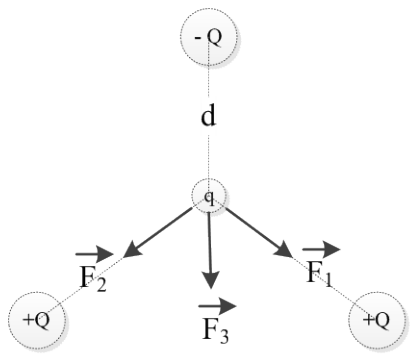


Solution

Step 1: Determine and draw the directions of the forces on q due to +Q and -Q:

Since q is negative, it is repelled by -Q but is attracted by +Q.
All forces are drawn along the lines connecting the centres of the particles. Repulsion forces are directed away from the repelling particle (-Q), whereas attractive forces are directed towards the attracting particles (+Q).

$\overrightarrow{F_1}$ is due to the +Q on the right.
$\overrightarrow{F_2}$ is due to the +Q on the left.
$\overrightarrow{F_3}$ is due to the –Q on the top.



Step 2: Calculate the magnitude of the forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, and $\overrightarrow{F_3}$.

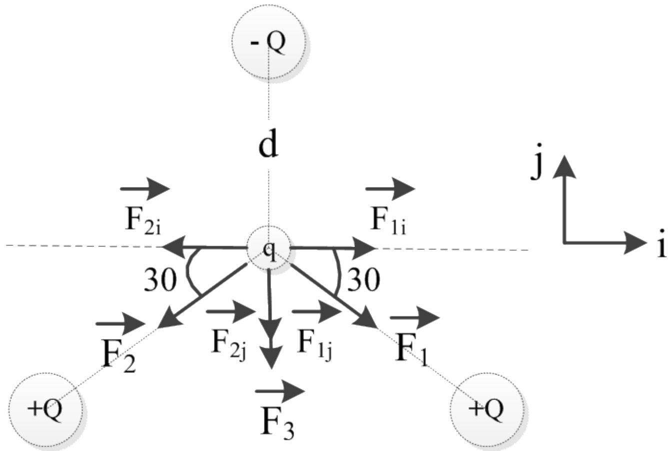
In general, $\left|\vec{F}\right| = \frac{kq_1q_2}{r^2}$

In the case of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$: q_1 = q, q_2 = +Q, r = d and $\left|\overrightarrow{F_1}\right| = \left|\overrightarrow{F_2}\right| = \frac{kqQ}{d^2}$

In the case of $\overrightarrow{F_3}$: q_1 = q, q_2 = -Q, r = d and $\left|\overrightarrow{F_3}\right| = \frac{kqQ}{d^2}$

Note: while the magnitudes of all forces are the same, their directions are different. We must, therefore, define vector directions and incorporate those into the forces to turn them into vector quantities.

Step 3: Introduce vector directions and break down forces into components



$\overrightarrow{F_1} = |\overrightarrow{F_1}|\, cos(30)i - |\overrightarrow{F_1}|\, sin(30)j$
$\overrightarrow{F_2} = -|\overrightarrow{F_2}|\, cos(30)i - |\overrightarrow{F_2}|\, sin(30)j$
$\overrightarrow{F_3} = -|\overrightarrow{F_3}|\, j$

Note that:
$\overrightarrow{F_{2i}} = -\overrightarrow{F_{1i}}$ *and both have a magnitude of* $|\overrightarrow{F_1}|\, cos(30)$.
$\overrightarrow{F_{2j}} = \overrightarrow{F_{1j}}$ *and both have a magnitude of* $|\overrightarrow{F_1}|\, sin(30)$, and the vectors are overlapping.

Step 4: Sum up the vector forces to obtain a net force

$$\overrightarrow{F_{net}} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = |\overrightarrow{F_1}|cos(30)i - |\overrightarrow{F_1}|sin(30)j - |\overrightarrow{F_2}|cos(30)i - |\overrightarrow{F_2}|sin(30)j - |\overrightarrow{F_3}|j$$

Group vectors based on their directions:

$$\overrightarrow{F_{net}} = \left[|\overrightarrow{F_1}|cos(30) - |\overrightarrow{F_2}|cos(30)\right] i - \left[|\overrightarrow{F_1}|sin(30) + |\overrightarrow{F_2}|sin(30) + |\overrightarrow{F_3}|\right] j$$

Since $|\overrightarrow{F_1}| = |\overrightarrow{F_2}| = |\overrightarrow{F_3}|$ this reduces to: $\overrightarrow{F_{net}} = [0]\, i - |\overrightarrow{F_1}|[2\, sin(30) + 1]\, j$

Step 5: Plug in values:

Change to SI units: 1 cm → 10^{-2} m, $|\overrightarrow{F_1}| = \frac{kqQ}{d^2} = \frac{9*10^9*1*10^{-6}*2*10^{-6}}{(1*10^{-2})^2} = 180$ (N)
$\overrightarrow{F_{net}} = -180 * [2 * \sin(30) + 1]j = -360\, j$ (N)

The net force on the negative charge q is 360 N pointing downwards.