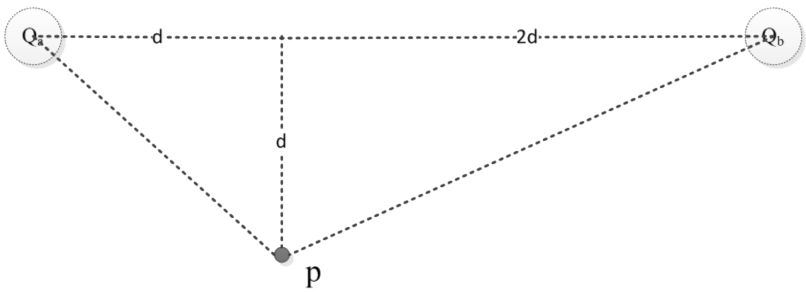
Field Hard Example (determine the electric field due to two charges):

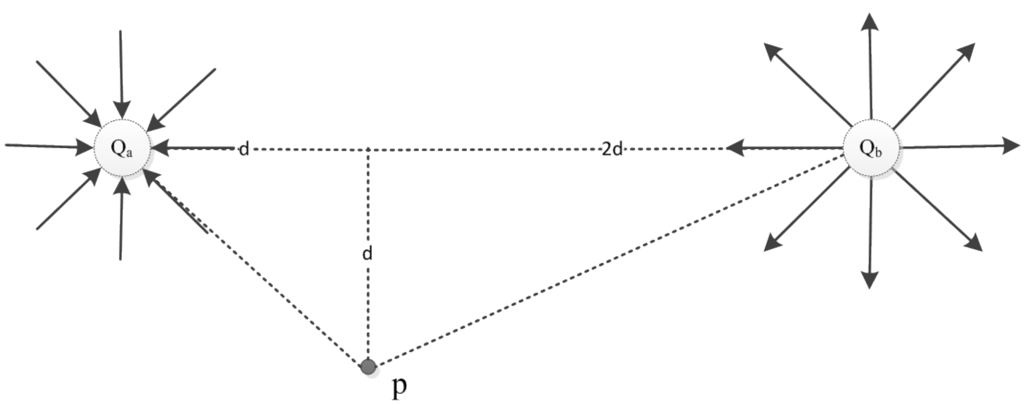
Determine the magnitude and direction of the electric field vector at point p.
Given: Q_a = -1 C, Q_b = 3 C, d = 1 cm and $k = 9 * 10^9 \ Nm^2/C^2$.



<u>Solution</u>

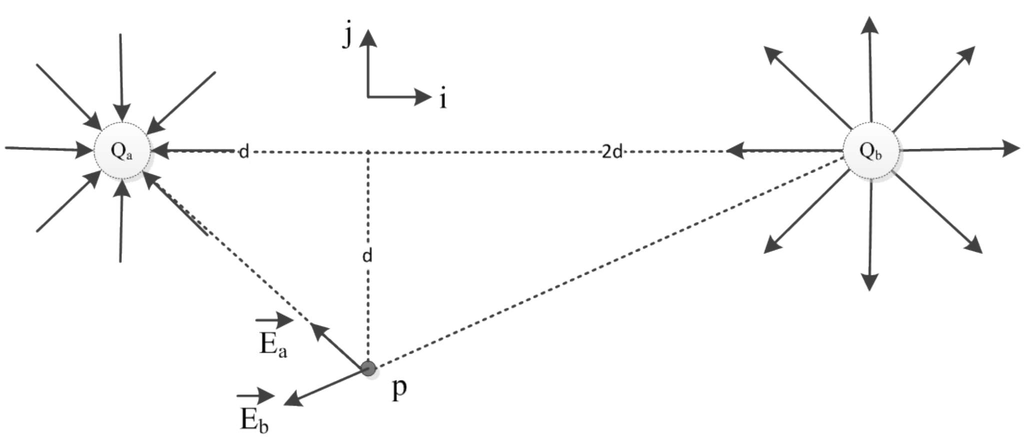
Step 1: Determine the direction of the electric field due to each charge separately.

Recall that the electric field is pointing away from the centre of a positive charge, but towards the centre of a negative charge. The electric fields are illustrated in the diagram below:



Step 2: Determine the direction of the electric field at point p.

The electric field vector must be directed along the line connecting p and the centres of each charge, i.e. along the dotted lines. The direction of each arrow - towards or away from the charge - is determined by the type of charge, as explained in the previous step.



Step 3: Calculate the magnitude of each electric field at point p.

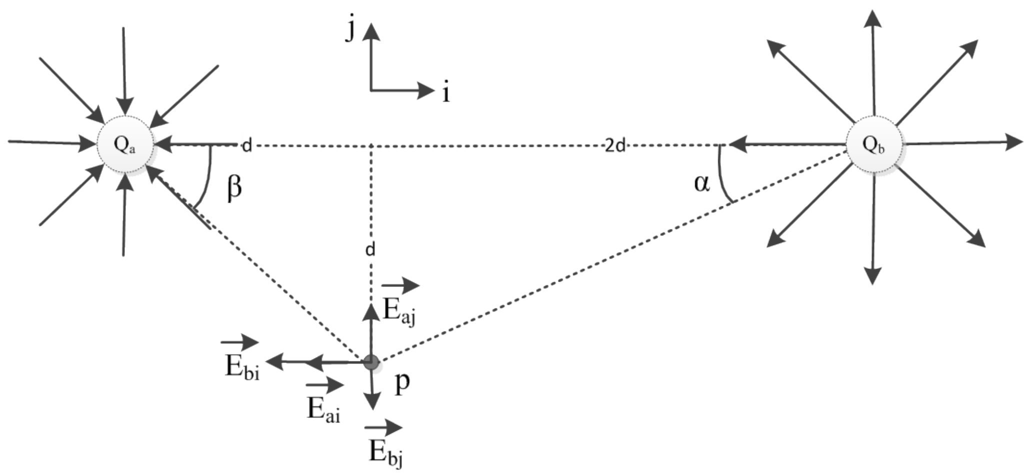
To apply the electric field equation $\left|\vec{E}\right| = \frac{kQ}{r^2}$ we must first determine the values of r_a and r_b which are measured from point p to the centre of charges Q_a and Q_b, respectively.

From geometry we see that: $r_a = \sqrt{d^2 + d^2} = \sqrt{2}d$ and $r_b = \sqrt{d^2 + (2d)^2} = \sqrt{5}d$

Plugging these into the above equation:

$\left|\overrightarrow{E_a}\right| = \frac{kQ_a}{{r_a}^2} = \frac{kQ_a}{(\sqrt{2}d)^2} = \frac{kQ_a}{2d^2}$ and $\left|\overrightarrow{E_b}\right| = \frac{kQ_b}{{r_b}^2} = \frac{kQ_b}{(\sqrt{5}d)^2} = \frac{kQ_b}{5d^2}$

Step 4: Decompose vectors into x and y components using unit vectors i and j respectively.



$tan(\beta) = \frac{d}{d} = 1 \;\Rightarrow\;$ ß $= tan^{-1}(1) = 45^o$
$tan(\alpha) = \frac{d}{2d} = \frac{1}{2} \;\Rightarrow\; \alpha = tan^{-1}(1/2) = 26.6^o$

$\left|\overrightarrow{E_{ai}}\right| = \left|\overrightarrow{E_a}\right|cos(\beta)$ and $\left|\overrightarrow{E_{aj}}\right| = \left|\overrightarrow{E_a}\right|sin(\beta)$
$\left|\overrightarrow{E_{bi}}\right| = \left|\overrightarrow{E_b}\right|cos(\alpha)$ and $\left|\overrightarrow{E_{bj}}\right| = \left|\overrightarrow{E_b}\right|sin(\alpha)$

Therefore:
$\overrightarrow{E_a} = -\left|\overrightarrow{E_a}\right|cos(\beta)\; i + \left|\overrightarrow{E_a}\right|sin(\beta)\; j$
$\overrightarrow{E_b} = -\left|\overrightarrow{E_b}\right|cos(\alpha)\; i - \left|\overrightarrow{E_b}\right|sin(\alpha)\; j$

Step 5: Sum up the electric field vectors to obtain a net field at point p.

$\overrightarrow{E_{Total}} = \overrightarrow{E_a} + \overrightarrow{E_b} = -\left|\overrightarrow{E_a}\right|cos(\beta)\; i + \left|\overrightarrow{E_a}\right|sin(\beta)\; j - \left|\overrightarrow{E_b}\right|cos(\alpha)\; i - \left|\overrightarrow{E_b}\right|sin(\alpha)\; j$

Group i and j components:

$\overrightarrow{E_{Total}} = -\left[\left|\overrightarrow{E_a}\right|cos(\beta) + \left|\overrightarrow{E_b}\right|cos(\alpha)\right] i + \left[\left|\overrightarrow{E_a}\right|sin(\beta) - \left|\overrightarrow{E_b}\right|sin(\alpha)\right] j$

Plug in the magnitudes of the electric fields:

$$\overrightarrow{E_{Total}} = -\left[\frac{kQ_a}{2d^2}cos(\beta) + \frac{kQ_b}{5d^2}cos(\alpha)\right]\ i + \left[\frac{kQ_a}{2d^2}sin(\beta) - \frac{kQ_b}{5d^2}sin(\alpha)\right]\ j$$

Plug in the numerical values:

$$\overrightarrow{E_{Total}} = \frac{9 * 10^9}{(10^{-2})^2}\left\{-\left[\frac{1}{2}cos(45) + \frac{3}{5}cos(26.6)\right]\ i + \left[\frac{1}{2}sin(45) - \frac{3}{5}sin(26.6)\right]\ j\right\}$$

$$\overrightarrow{E_{Total}} = -8.01 * 10^{13}\ i + 7.64 * 10^{12}\ j\ \text{(N/C)}$$

Note: The total electric field at point p is pointing in the negative x and positive y directions.
Its magnitude is: $\left|\overrightarrow{E_{Total}}\right| = \sqrt{(8.01)^2 + (0.764)^2} * 10^{13} = 8.05 * 10^{13}$ (N/C)