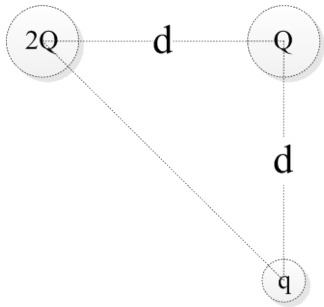
**Force Easy Example (determine the force due to two charges using Coulomb's law):**

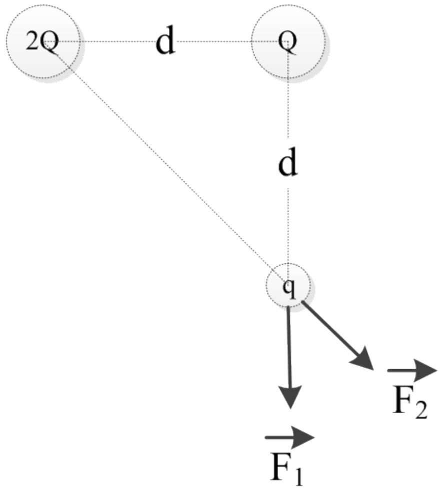
Calculate the forces experienced by the positive charge q due to each of the charges +Q and +2Q in the following diagram:




<u>Solution</u>

Step 1: Determine and draw the directions of the forces on q due to Q and 2Q.

Since all particles are positive, particle q is repelled by both Q and 2Q.
Repulsion forces are directed along the line connecting the centres of the particles, and away from the repelling particle. As a result, both $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are directed away from Q and 2Q, respectively.



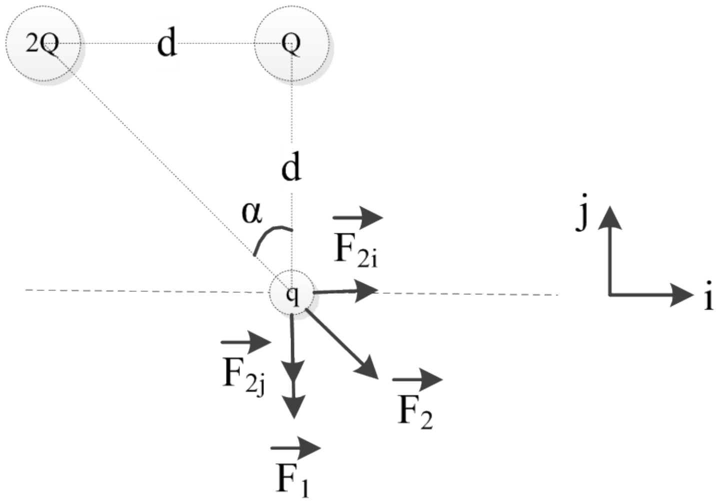

Step 2: Calculate the magnitude of the forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ due to the interaction between q and Q, and q and 2Q, respectively.

In general, $\left|\vec{F}\right| = \frac{kq_1q_2}{r^2}$

In the case of $\overrightarrow{F_1}$ : $q_1 = q$, $q_2 = Q$, and $r = d$: $\left|\overrightarrow{F_1}\right| = \frac{kqQ}{d^2}$

In the case of $\overrightarrow{F_2}$ : $q_1 = q$, $q_2 = 2Q$, and $r = \sqrt{2}d$: $\left|\overrightarrow{F_2}\right| = \frac{kq2Q}{2d^2} \Rightarrow \left|\overrightarrow{F_2}\right| = \frac{kqQ}{d^2} \Rightarrow \left|\overrightarrow{F_2}\right| = \left|\overrightarrow{F_1}\right|$

Step 3: Introduce vector directions and break down forces into components.




$tan(\alpha) = \frac{d}{d} = 1 \ \Rightarrow \ \alpha = tan^{-1}(1) = 45^o$

$\left|\overrightarrow{F_{2i}}\right| = \left|\overrightarrow{F_2}\right| sin(\alpha)$

$\left|\overrightarrow{F_{2j}}\right| = \left|\overrightarrow{F_2}\right| cos(\alpha)$

$$\overrightarrow{F_2} = \left|\overrightarrow{F_2}\right| [sin(\alpha)i - cos(\alpha)j] = \left|\overrightarrow{F_2}\right| [sin(45)i - cos(45)j] = \left|\overrightarrow{F_2}\right| \left[\frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j\right]$$

$\overrightarrow{F_1} = -\left|\overrightarrow{F_1}\right| j$

Step 4: Sum up the vector forces to obtain a net force.

$$\overrightarrow{F_{net}} = \overrightarrow{F_1} + \overrightarrow{F_2} = -\left|\overrightarrow{F_1}\right| j + \left|\overrightarrow{F_2}\right| \left[\frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j\right] = \left[\left|\overrightarrow{F_2}\right| \frac{1}{\sqrt{2}}\right] i - \left[\left|\overrightarrow{F_1}\right| + \left|\overrightarrow{F_2}\right| \frac{1}{\sqrt{2}}\right] j$$

Step 5: Plug in the expressions for $\left|\overrightarrow{F_1}\right|$ and $\left|\overrightarrow{F_2}\right|$.

$$\overrightarrow{F_{net}} = \left[\frac{kqQ}{d^2}\frac{1}{\sqrt{2}}\right] i - \left[\frac{kqQ}{d^2} + \frac{kqQ}{d^2}\frac{1}{\sqrt{2}}\right] j = \frac{kqQ}{d^2}\left[\frac{1}{\sqrt{2}} i - \left(1 + \frac{1}{\sqrt{2}}\right) j\right] = \frac{kqQ}{d^2}(0.707\ i - 1.707\ j)$$